Comment on "Non-Mean-Field Behavior of the Contact Process on Scale-Free Networks"

Recently, Castellano and Pastor-Satorras [1] used the finite size scaling (FSS) theory to analyze simulation data for the contact process (CP) on scale-free networks (SFNs) and claimed that its absorbing critical behavior is not consistent with the mean-field (MF) prediction. Furthermore, they pointed out large density fluctuations at highly connected vertices as a possible origin for non-MF critical behavior. In this Comment, we propose a scaling theory for relative density fluctuations in the spirit of the MF theory, which turns out to explain simulation data perfectly well. We also measure the value of the critical density decay exponent $\beta/\nu_{\parallel}$, which agrees well with the MF prediction. Our results strongly support that the CP on SFNs still exhibits a MF-type critical behavior.

In contrast to equilibrium models with thermal noise, nonequilibrium models exhibiting absorbing critical phenomena involve so-called multiplicative noise. In particular, the noise amplitude is proportional to the square root of the particle density in models belonging to the directed percolation class such as CP [2]. Therefore, at the MF level neglecting spatial fluctuations, the particle number fluctuation should be proportional to the square root of the total number of particles. It is easy to derive the relative density fluctuation on vertices of degree k as

$$r_k = \Delta\rho_k/\rho_k \sim [N\rho_k P_k]^{-1/2}, \quad (1)$$

where ρ_k is the partial density on vertices of degree k, N is the total number of vertices, and P_k is the degree distribution. As $\rho_k \sim \rho k$ near criticality (ρ: total density), we find, for reasonably large k,

$$r_k \sim (N\rho)^{-1/2} k^{(\gamma-1)/2}, \quad (2)$$

where γ is the degree exponent.

For large enough $k > N^{1/\gamma}$ where the degree distribution becomes almost flat in practice [$NP_k \sim O(1)$], we expect $r_k \sim (\rho k)^{-1/2}$. With these two limiting results and also with $\rho \sim N^{-\alpha}$ at criticality, one may assume a critical scaling relation as

$$r_k = N^{(\alpha-1/\gamma)/2} f(kN^{-1/\gamma}), \quad (3)$$

where $f(x) \sim x^{(\gamma-1)/2}$ as $x \to 0$ and $f(x) \sim x^{-1/2}$ as $x \to \infty$. In Fig. 1, our simulation results at criticality for $\gamma = 2.75$ and 4.0 with various sizes $N = 10^3, \ldots, 10^6$ show excellent agreement with our prediction. Note that the case of $\gamma = 4.0$ where the γ-independent ordinary MF appears also shows the similar behavior in r_k.

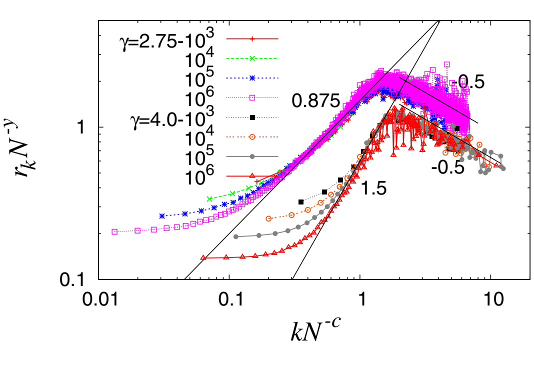

FIG. 1 (color online). Data collapse of the relative density fluctuations r_k averaged over surviving samples in the steady state for $\gamma = 2.75$ ($p_c = 0.4240$, $c = 0.363$, $y = 0.104$) and $\gamma = 4.0$ ($p_c = 0.3570$, $c = 0.25$, $y = 0.125$), respectively.

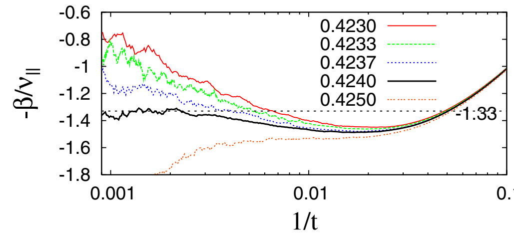

FIG. 2 (color online). Effective exponents $\beta/\nu_{\parallel}$ for $\gamma = 2.75$ with $N = 10^7$. All samples are alive when data are obtained.

We also observe the density decay dynamics for $\gamma = 2.75$ in Fig. 2, where the effective decay exponents $\beta/\nu_{\parallel}$ are plotted against $1/t$. Our best estimate is $\beta/\nu_{\parallel} = 1.34(5)$, which agrees well with the MF prediction of $\nu_{\parallel} = 1$ and $\beta = 1/(\gamma - 2)$ for $\gamma < 3$.

Finally, we comment on the FSS exponent $\alpha = \beta/\bar{\nu}$. Vertices of degree $k > N^{1/\gamma}$ are practically irrelevant. Accordingly, the cutoff in k bigger than $N^{1/\gamma}$ cannot influence critical FSS which should be determined by the competition between correlated volume and system size. We conjecture that $\bar{\nu} = (\gamma - 1)/(\gamma - 2)$ from the hyperscaling-type argument [3]. Our numerical estimate $\beta/\bar{\nu} = 0.59(2)$ for $\gamma = 2.75$ agrees well with our conjecture. In fact, the results of Ref. [1] also agree reasonably well with our conjecture. In conclusion, we believe that the CP on SFNs exhibits the MF-type critical behavior.

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