## arXiv:2204.07388v3 [cond-mat.stat-mech] 16 Jun 2022

## Speed limit for a highly irreversible process and tight finite-time Landauer's bound

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(Dated: June 17, 2022)

Landauer's bound is the minimum thermodynamic cost for erasing one bit of information. As this bound is achievable only for quasistatic processes, finite-time operation incurs additional energetic costs. We find a "tight" finite-time Landauer's bound by establishing a general form of the classical speed limit. This tight bound well captures the divergent behavior associated with the additional cost of a highly irreversible process, which scales differently from a nearly irreversible process. We also find an optimal dynamics which saturates the equality of the bound. We demonstrate the validity of this bound via discrete one-bit and coarse-grained bit systems. Our work implies that more heat dissipation than expected occurs during high-speed irreversible computation.

Introduction – Memory erasure is an elementary operation in irreversible computation. As the erasing operation incurs a thermodynamic cost and takes a finite physical time, low energy consumption and a short process time are critical requirements for efficient computation. The fundamental limitation of the energetic cost is given by Landauer's principle [1, 2], which states that at least  $k_{\rm B}T \ln 2$  of work is necessary to erase a single bit memory, where  $k_{\rm B}$  is the Boltzmann constant and T is the environment temperature. Landauer's bound is universal in the sense that it is independent of memory device type or physical platform. This bound has been confirmed experimentally using various physical setups, including a double-well potential realized by optical tweezers [3, 4]and virtual potential [5, 6], an electric-circuit system [7], and a nanomagnetic memory bit [8, 9].

In real-world situations, however, this fundamental bound is less practical as it requires a quasistatic process, which far exceeds the system's relaxation time scale. The reported experimental time scale of the "quasistatic" erasure process ranges from a few hundred milliseconds [10] to several tens [4] or hundreds [5] of seconds, which is far from the time required for practical computation. Therefore, it is important to understand the finite-time effect on thermodynamic cost, which generally increases as a process becomes faster and more irreversible [11– 15]. Several experimental studies have suggested that the minimum energetic cost should increase by an additional cost inversely proportional to the erasing time  $\tau$ , i.e.,  $k_{\rm B}T \ln 2 + C/\tau$ , with a system-dependent constant C [3–6]. This behavior has also been investigated theoretically for the classical stochastic system described by the overdamped Langevin equation [16, 17], and an open quantum system described by the Lindblad equation [18].

These studies suggest that a trade-off relation plays a central role in understanding the overhead cost of the Landauer's bound. Over the last decade, various types of trade-off relations have been reported in stochastic systems and also for open quantum systems such as thermodynamic uncertainty relations [19–27], kinetic uncertainty relations [28–32] and speed limits for state change [33–42]. Recently, Zhen et al. [43] showed that the  $1/\tau$  behavior of the "minimum work bound" of erasing processes is governed by the speed-limit inequality associated with the thermodynamic cost.

In this Letter, we first present a simpler way to derive the general form of the speed limit introduced in Ref. [42], which can have various functional forms. Two different speed limit regimes are considered in terms of the degree of irreversibility. For a nearly reversible process, we retrieve the previous speed limits [33, 34] by taking a simple functional form, that provide a tight bound on the operation time in terms of entropy production (EP) and dynamical activity. However, this bound gradually loosens as the process becomes more irreversible. We find a tight bound for a highly reversible process from the general speed limit with a different functional form. In the limit of high irreversibility, this new bound becomes finite, depending solely on dynamical activity.

We apply this general speed limit to calculate the tight bound of the additional cost associated with a finitetime erasing operation. We demonstrate that this bound scales as  $1/\tau$  for a nearly reversible process, while much stronger divergence appears for a fast or highly irreversible process. As a result, high-speed irreversible computation requires much more heat dissipation and thus associated supporting cooling architecture. We also find an explicit protocol which meets the equality condition of the bound.

Speed limit for a highly irreversible process – Suppose that the time evolution of a probability distribution  $p(t) = \{p_n(t)\}$  at time t is described by

$$\dot{p}_n(t) = \sum_m R_{nm}(t) p_m(t),$$

where the transition rate matrix  $\mathbf{R}(t)$  satisfies the condition  $R_{nm}(t) \geq 0$  for  $m \neq n$  and  $R_{nn}(t) = -\sum_{m(\neq n)} R_{mn}(t)$ . The statistical distance between the initial and the final distributions after time  $\tau$  can be mea-

sured by the total variational distance

$$\ell = d_T(\boldsymbol{p}(\tau), \boldsymbol{p}(0)) \equiv \frac{1}{2} \sum_n |p_n(\tau) - p_n(0)|.$$

We first establish a general form of the speed limit, in terms of the distance  $\ell$ , the EP  $\Sigma^*$ , and the total activity (number of jumps)  $A_{\text{tot}}$  during  $\tau$ , given by

$$\frac{\ell}{A_{\rm tot}} \le f\left(\frac{\Sigma^*}{A_{\rm tot}}\right),\tag{1}$$

with an appropriate choice of a monotonically increasing concave function f as listed in Table I. Hereafter, we set  $k_{\rm B} = 1$  for convenience. The EP  $\Sigma^* = \int_0^{\tau} (\dot{\Sigma}^*) dt$ is characterized by the transition matrix of the adjoint process  $\mathbf{R}^*(t)$  [44–47], where the corresponding EP rate is defined as

$$\dot{\Sigma}^* \equiv \sum_{n \neq m} R_{nm}(t) p_m(t) \ln \left[ \frac{R_{nm}(t) p_m(t)}{R_{mn}^*(t) p_n(t)} \right].$$
(2)

We assume that the adjoint process is also stochastic with the same escape rate;  $R_{nn}^*(t) = -\sum_{m(\neq n)} R_{mn}^*(t) =$  $R_{nn}(t)$ . A trivial choice of the adjoint process is taking it the same as the time-reversal process,  $R_{mn}^*(t) = R_{mn}(t)$ , which leads to the total EP  $\Sigma$ , i.e.  $\Sigma^* = \Sigma$ . Alternatively, by considering the instantaneous steady-state  $\boldsymbol{p}^{\rm ss}(t)$  such that  $\dot{\boldsymbol{p}}^{\rm ss}(t) = 0$ , the adjoint process defined as  $R_{mn}^*(t) = R_{nm}(t) \left(\frac{p_{m}^{\rm ss}(t)}{p_{n}^{\rm ss}(t)}\right)$  leads to the Hatano-Sasa (excess) EP  $\Sigma^* = \Sigma_{\rm HS}$ . Since  $0 \leq \Sigma_{\rm HS} \leq \Sigma$  [44, 48],  $\Sigma_{\rm HS}$ always gives a tighter bound than  $\Sigma$  in Eq. (1).

To derive Eq. (1), we first note that

$$\ell = \frac{1}{2} \sum_{n} |p_n(\tau) - p_n(0)| \le \frac{1}{2} \int_0^\tau dt \sum_{n} |\dot{p}_n(t)|, \quad (3)$$

from the triangle inequality. Using  $R_{nn}^*(t) = R_{nn}(t)$ , the instantaneous change of the probability distribution is bounded by

$$\sum_{n} |\dot{p}_{n}(t)| = \sum_{n} \left| \sum_{m(\neq n)} R_{nm}(t) p_{m}(t) + R_{nn}(t) p_{n}(t) \right|$$
$$= \sum_{n} \left| \sum_{m(\neq n)} \left\{ R_{nm}(t) p_{m}(t) - R_{mn}^{*}(t) p_{n}(t) \right\} \right|$$
$$\leq \sum_{n \neq m} |R_{nm}(t) p_{m}(t) - R_{mn}^{*}(t) p_{n}(t)|$$
$$= 2A(t) d_{T}(\boldsymbol{Q}(t), \boldsymbol{Q}^{*}(t)), \qquad (4)$$

where  $A(t) = \sum_{n \neq m} R_{nm}(t) p_m(t) = \sum_{m \neq n} R_{mn}^*(t) p_n(t)$ is a jump rate at time t and Q(t) ( $Q^*(t)$ ) denotes the normalized conditional joint probability distribution of the forward (reverse) process which is defined as follows:

$$\begin{aligned} Q_{mn}(t) &= P[m, n | \text{jump}] = \frac{(1 - \delta_{nm}) R_{nm}(t) p_m(t)}{A(t)}, \\ Q_{mn}^*(t) &= P^*[n, m | \text{jump}] = \frac{(1 - \delta_{mn}) R_{mn}^*(t) p_n(t)}{A(t)} . \end{aligned}$$

TABLE I. Various choices of the concave function f(x) satisfying Eq. (7) and its inverse (convex) function  $h(v) = f^{-1}(v)$ .

	f(x)	h(v)
Pinsker [49]	$\sqrt{(x/2)}$	$2v^2$
Bretagnolle–Huber [50]	$\sqrt{1-e^{-x}}$	$-\ln(1-v^2)$
Vajda [51]	n/a <sup>a</sup>	$\ln\left(\frac{1+v}{1-v}\right) - \frac{2v}{1+v}$
Gilardoni [52]	n/a $^{\rm a}$	$\ln\left(\frac{1+v}{1-v}\right) - \frac{2v}{1+v}$ $\ln\left[\frac{(1+v)^{-1+v}}{1-v}\right]$
Symmetric KLD $^{\rm b}$ [53, 54]	n/a $^{\rm a}$	$v \ln \left[\frac{1+v}{1-v}\right]^{2}$

<sup>a</sup> Analytic compact expression is not available.

<sup>b</sup> This bound is valid only when the KLD is symmetric.

Thus,  $d_T(\mathbf{Q}(t), \mathbf{Q}^*(t))$  captures how much irreversible the process is at time t. We remark that  $A_{\text{tot}} = \int_0^{\tau} dt A(t)$ has a meaning of the total number of jumps during the entire process. By combining Eqs. (3) and (4), we have the following inequality:

$$\ell \leq \int_0^\tau dt A(t) d_T(\boldsymbol{Q}(t), \boldsymbol{Q}^*(t)).$$
(5)

It is worth nothing that the EP rate can be expressed in terms of the conditional joint distributions as

$$\dot{\Sigma}^* = A(t)D(\boldsymbol{Q}(t)||\boldsymbol{Q}^*(t)), \qquad (6)$$

where  $D(\boldsymbol{p}||\boldsymbol{q}) \equiv \sum_{x} p_x \ln(p_x/q_x)$  is the Kullback-Leibler divergence (KLD) between two probability distributions  $\boldsymbol{p}$  and  $\boldsymbol{q}$ . Note that the KLD corresponding to the total EP rate ( $\dot{\Sigma}$ ) is symmetric ( $D(\boldsymbol{p}||\boldsymbol{q}) = D(\boldsymbol{q}||\boldsymbol{p})$ ), while the KLD is generally asymmetric for other choices such as  $\dot{\Sigma}_{\text{HS}}$ . There exist various choices of a monotonic concave function f (see Table I) that connects the total variational distance and the KLD to obey the following inequality [49–52]:

$$d_T(\boldsymbol{p}, \boldsymbol{q}) \le f(D(\boldsymbol{p}||\boldsymbol{q})). \tag{7}$$

The speed limit is obtained by plugging in Eqs. (6) and (7) to Eq. (5), and then dividing both sides with  $A_{\text{tot}}$ , which leads to

$$\frac{\ell}{A_{\text{tot}}} \le \frac{\int_0^\tau dt A(t) f\left(\frac{\dot{\Sigma}^*}{A(t)}\right)}{\int_0^\tau dt A(t)} \le f\left(\frac{\Sigma^*}{A_{\text{tot}}}\right), \qquad (8)$$

from the concavity of f.

As  $g(x) \equiv h(x)/(2x)$  is a monotonically increasing function for all h's in Table I, where  $h = f^{-1}$ , we can rewrite Eq. (8) as

$$\tau \ge \frac{\ell}{\langle A \rangle_{\tau} g^{-1} \left(\frac{\Sigma^*}{2\ell}\right)},\tag{9}$$

by defining  $\langle A \rangle_{\tau} \equiv A_{\text{tot}}/\tau$  and  $g^{-1}(x)$  the inverse function of g(x). Equation (9) is the general form of the speed limit, where various types of bounds can be obtained based on the choice of h(x). The previous speed limit  $\tau \geq 2\ell^2/(\langle A \rangle_{\tau} \Sigma^*)$  in Refs. [33, 34] is readily obtained by taking  $h(x) = x^2$  (Pinsker [49]), which is tight only for a nearly reversible (slow) process but yields a very loose bound for a highly irreversible process. We note that for any  $\Sigma^* \geq 2\ell$ , this bound is even worse than the *fundamental* bound  $\tau \geq \ell/\langle A \rangle_{\tau}$  obtained from the minimum activity to change the probability distribution regardless of the EP,  $A_{\text{tot}} = \langle A \rangle_{\tau} \tau \geq \ell$  [42].

We find that a speed limit can be tightened for a highly irreversible process with alternative choices of h(x) such as Bretagnolle–Huber, Vajda, Gilardoni, and symmetric KLD as listed in Table I. All these four functions provide speed limits, always tighter than the fundamental bound, which can be accessible only when  $\Sigma^* \to \infty$ . Therefore, in the highly irreversible limit, time is bounded solely by the dynamical activity, but not the EP. The symmetric KLD bound is always the tightest among all h(x), though it is valid only for the symmetric KLD. Otherwise, the Gilardoni bound is the tightest for  $\Sigma^*/\ell \ge 1.14$ , while the Pinsker bound is the tightest elsewhere. Simple derivation of the symmetric KLD bound is presented in Supplemental Material (SM) [55].

Tight finite-time Landauer's bound – The speed limit in Eq. (1) can be rearranged to bound the EP as

$$\Sigma^* \ge \ell \frac{h(v)}{v} \equiv B_H, \tag{10}$$

where  $v = \ell/A_{\text{tot}}$  is the average distance change per jump, which ranges from 0 to 1, measuring the irreversibility of the process. When v is close to 0 (1), the distribution changes gradually (abruptly), so the process is nearly reversible (highly irreversible). The bound  $B_H$ monotonically increases with v for all h's, where H denotes a specific functional form, e.g., H = P (Pinsker) and H = S (symmetric KLD) with  $B_P = 2\ell v$  and  $B_S = 2\ell \tanh^{-1} v$ .

Now, we use the EP bound to estimate the minimum cost for a finite-time erasing process. Suppose an erasing operation resets a one-bit system composed of 0 and 1 states with the associated probabilities  $p_0(t)$  and  $p_1(t)$ , respectively. Let us assume that the initial bit is random with probability distribution as  $(p_0(0), p_1(0)) =$ (1/2, 1/2), and the erasing process yields the final distribution  $(p_0(\tau), p_1(\tau)) = (1 - \epsilon, \epsilon)$  with erasing error  $\epsilon$  after time  $\tau$ . The statistical distance between the initial and final states becomes  $\ell = 1/2 - \epsilon$ , and the Shannon entropy change of the system can be computed as  $\Delta S_{\text{sys}} = -\ln 2 - (1 - \epsilon) \ln(1 - \epsilon) - \epsilon \ln \epsilon$ . Furthermore, by setting  $\Sigma^*$  as the total EP [56], we have  $\Sigma^* = \Delta S_{\rm sys} + Q/T$ , where Q is the heat dissipated into the surrounding environment with temperature T during the erasing process. As the total EP  $\Sigma$  corresponds to the symmetric KLD, we use the symmetric KLD bound

to obtain the tightest Landauer's bound.

In the perfect erasing limit  $\epsilon \to 0$ , we get  $\ell \to 1/2$  and  $\Delta S_{\rm sys} \to -\ln 2$  and the finite-time Landauer's bound from Eq. (10) is expressed as

$$\frac{Q}{T} \ge \ln 2 + B_{\rm S} = \ln 2 + \tanh^{-1} v,$$
 (11)

where  $B_{\rm S}$  represents the additional cost due to finitetime operation. For small v (nearly reversible),  $B_{\rm S} \simeq v = 1/(2\tau \langle A \rangle_{\tau})$ , which corresponds to the previously known  $1/\tau$  behavior [3–6, 43]. As we approach  $v = 1^-$  (highly irreversible regime),  $B_{\rm S}$  diverges asymptotically as

$$B_{\rm S} \simeq -\frac{1}{2}\ln\left(1-v\right) = -\frac{1}{2}\ln\left(1-\frac{1}{2\tau\langle A\rangle_{\tau}}\right). \tag{12}$$

This implies that much higher dissipation should occur in a highly irreversible erasing operation.

Practical computation requires a small erasing error as well as a short operation time. To this end, the transition rate from 1 to 0 state has to be large for fast operation, necessitating large driving (large  $\langle A \rangle_{\tau}$ ). In comparison, the reverse transition (0 to 1) should be suppressed to prevent erasing-error operations. Therefore, the best strategy for a desired erasing operation is that all "particles" initially located at state 1 jump to state 0 once, and no jump occurs afterwards; this condition can be read as  $A_{\text{tot}} \approx 1/2$  with  $p_1(0) = 1/2$ . Consequently, the operation for a practical erasing process should be highly irreversible with  $v = \ell/A_{\text{tot}} \approx 1$ . Thus, Eq. (12) for a highly irreversible process is well deserved for practical computation.

We find explicitly the *optimal* dynamics which minimizes the EP, satisfying the equality of Eq. (10) with  $h = h_{\rm S}$ . Its sufficient condition is

$$\frac{R_{01}(t)p_1(t)}{R_{10}(t)p_0(t)} = c \text{ (const.)}, \quad \forall \ 0 \le t \le \tau \ , \tag{13}$$

along with monotonic change of  $p_n(t)$  in time. The detailed derivation is presented in SM [55], where we also show that a process with  $v \approx 1^-$  ( $v \approx 0$ ) is realized with large c and short  $\tau$  ( $c \approx 1$  and long  $\tau$ ).

The finite-time Landauer's bound, Eqs. (10) and (11), is also applicable to a bit system made by coarsening, such as a Langevin system with a double-well potential [3–6]. This can be verified by the fact that the EP of a coarse-grained bit system is equal to or smaller than that of its original system without coarse-graining. See Eq. (10) in Ref. [43] and SM [55] for details. Therefore, the original EP is also bounded by the same additional cost term in Eq. (10).

Numerical confirmation – Here we investigate two examples. The first one is a discrete one-bit system consisting of states 0 and 1 with energy levels  $E_0 = 0$  (fixed) and  $E_1(t)$  (time-varying), respectively. Its dynamics is



FIG. 1. (a, b) Schematics of the two erasure models: discrete one-bit system (a) and coarse-grained bit system (b). (c) Plot of  $\Sigma/\ell$  versus  $v^{-1}$  for discrete system (gray ×) and continuous system (green •). The orange solid (sky-blue dashed) curve denotes the Pinsker (symmetric KLD) bound. The result of the optimal protocol is denoted by red dot •. (d) Plot of  $\Sigma$  divided by the the symmetric KLD bound  $B_{\rm S}$  versus  $v^{-1}$ . For a fixed  $\ell$  and A(t),  $v^{-1}$  can be simply regarded as a scaled time. The same data are used for (c) and (d).

described by the following master equation:

$$\dot{p}_0(t) = R_{01}(t)p_1(t) - R_{10}(t)p_0(t),$$
  
$$\dot{p}_1(t) = R_{10}(t)p_0(t) - R_{01}(t)p_1(t).$$
(14)

The transition rate  $R_{nm}(t)$  satisfies the detailed balance condition, that is, with  $\gamma(t) \equiv e^{-E_1(t)/T}/(1+e^{-E_1(t)/T})$ 

$$R_{10}(t) = \mu(t)\gamma(t), \quad R_{01}(t) = \mu(t)(1 - \gamma(t)), \quad (15)$$

where  $\mu(t)$  is an overall transition rate. Erasing process is illustrated in Fig. 1(a). The system is prepared with the

initial distribution  $(p_0, p_1) = (1/2, 1/2)$  with  $E_1 = 0$  and  $\mu = 0$  for t < 0. Here,  $\mu = 0$  indicates that the transition is blocked.  $E_1$  and  $\mu$  are abruptly raised to  $E_{\text{eras}}$  and  $\mu_{\text{eras}}$  at time t = 0, respectively, and maintained up to  $t = \tau$ . And then, both  $E_1$  and  $\mu$  are immediately lowered to 0 at  $t = \tau$ . The final distribution at  $t = \tau$  is  $(p_0, p_1) = (1 - \epsilon, \epsilon)$ . This protocol is the simplest one in the *n*-step energy-raising procedure [43, 57]. The exact solution of this model is

$$p_1(t) = e^{-\mu t} p_1(0) + (1 - e^{-\mu t}) p_{1, E_{\text{eras}}}^{\text{eq}}, \qquad (16)$$

where  $p_{1,E_{\text{eras}}}^{\text{eq}} = 1/[1 + \exp(E_{\text{eras}}/T)]$ . Using Eq. (16), we explicitly calculate the entropy change of the system  $\Delta S_{\text{sys}} = -\sum_i [p_i(\tau) \ln p_i(\tau) - p_i(0) \ln p_i(0)]$  and heat  $Q = E_{\text{eras}}[p_1(\tau) - p_1(0)]$ , which leads to the total EP  $\Sigma = \Delta S_{\text{sys}} + Q/T$ , as well as the total activity  $A_{\text{tot}}$ . We also construct an optimal time-dependent control of  $R_{nm}(t)$ satisfying the saturation condition Eq. (13), of which the explicit form can be found in SM [55].

The second example is a coarse-grained bit system consisting of a one-dimensional Brownian particle trapped in a double-well potential. Dynamics of the particle is governed by the following overdamped Langevin equation:

$$\gamma \dot{x} = -\frac{\partial V_{\rm DW}(x,t)}{\partial x} + \sqrt{2\gamma T} \xi(t), \qquad (17)$$

where x is position of the particle,  $\xi(t)$  is a Gaussian white noise satisfying  $\langle \xi(t)\xi(t')\rangle = \delta(t-t')$ , and the double-well potential  $V_{\text{DW}}(x,t)$  is given as

$$V_{\rm DW}(x,t) = E_b \left[ \left(\frac{x}{x_m}\right)^4 - 2\left(\frac{x}{x_m}\right)^2 \right] + \Theta(t)\frac{x}{x_m}, \quad (18)$$

where  $\Theta(t)$  provides a time-dependent protocol. This model corresponds to the experimental setup in Ref. [58]. The system can be treated as a coarse-grained bit memory by regarding the system being in state "0" ("1") when  $x \leq 0$  (x > 0). Then, the probabilities for the coarsegrained state i ( $i \in \{0, 1\}$ ) are

$$p_0^{\text{cg}}(t) = \int_{x \le 0} dx P(x, t) \text{ and } p_1^{\text{cg}}(t) = 1 - p_0^{\text{cg}}(t),$$
 (19)

where P(x,t) is the probability distribution of the original continuous system. The erasing process of this model is presented in Fig. 1(b). An initial state is prepared as the equilibrium state determined by the double-well potential with  $\Theta(t) = 0$  for t < 0. As the potential is symmetric with respect to x = 0,  $p_0^{\text{cg}}(0) = p_1^{\text{cg}}(0) =$ 1/2. At t = 0, we immediately raises  $\Theta(t)$  to  $\Theta_{\text{eras}}$ and maintain it up to  $t = \tau$ .  $\Theta(t)$  then returns to 0 at  $t = \tau$ . Due to the nonlinearity of the potential force, an analytic solution is not available. Instead, the total EP  $\Sigma = \Delta S_{\text{sys}} + Q/T$  is estimated by numerically evaluating  $\Delta S_{\text{sys}} = -\int dx [P(x,\tau) \ln P(x,\tau) -$   $P(x,0) \ln P(x,0)$ ] and  $Q = \int_0^{\tau} dt (-\partial_x V_{\rm DM}) \circ \dot{x}(t)$ , where  $\circ$  is the Stratonovich product.  $\ell$  and  $A_{\rm tot}$  are estimated by using  $p_i^{\rm cg}(t)$  and by counting the number of transitions between the different coarse-grained states [59].

Figure 1(c) shows the plot of  $\Sigma/\ell$  against  $v^{-1}$  for the discrete and the coarse-grained bit models. The data for the discrete model are obtained by varying parameters  $E_{\rm eras}$  and  $\tau$  within the ranges  $10^{-5} \leq E_{\rm eras} \leq 10$  and  $10^{-10} \leq \tau \leq 20$  with fixed  $\mu_{\rm eras} = 1$  and T = 1. The data of the coarse-grained bit model are the simulation results for the parameter ranges (used in real experiment [58]) of  $0.1k_{\rm B}T \leq \Theta_{\rm eras} \leq 10k_{\rm B}T$  and  $0.1 \text{ ms} \leq \tau \leq 110 \text{ ms}$  with fixed  $x_m = 50 \text{ nm}$ ,  $k_{\rm B}T = 4.1 \text{ pN} \cdot \text{nm} (T = 300 \text{ K})$ ,  $E_b = 3k_{\rm B}T$ , and  $\gamma = 24\sqrt{2}k_{\rm B}T \cdot \text{ms}/\pi x_m^2$ . Each point of the coarse-grained bit model in the plot is obtained by averaging  $10^6$  realizations. The Pinsker and the symmetric KLD bounds are presented along with the result of the optimal erasing process in the figure and the comparison with other bounds is shown in SM [55].

Indeed, the symmetric KLD tightly bounds the EP of the discrete bit model for all v. This tightness can be also checked in Fig. 1(d), which presents the total EP divided by  $B_{\rm S}$  (see Eq. (10)). Note that the Pinsker bound is quite tight for nearly-reversible processes (small v); however, it becomes extremely loose near v = 1. The data of the coarse-grained model are also well bounded by  $B_{\rm S}$ . However, the bound is not tight due to the "intra EP" induced by transitions between *microstates* inside the same coarse-grained state. The detailed explanation is presented in SM [55]. Thus, it is also important to reduce the intra EP for lowering the thermodynamic cost for a coarse-grained system.

Conclusion – We find the finite-time Landauer's bound, which is tight for an erasing process with any irreversiblility and any error rate, from the general form of the speed limit. We also find an optimal dynamics which saturates the equality of the bound. This bound is applicable to a coarse-grained bit system as well as an intrinsically two-state system. We demonstrate that, for a highly irreversible process, the diverging behavior of the additional cost is much steeper than that of a nearly reversible process. This indicates that, in a practical computation, which belongs to a highly irreversible regime, reducing the operation time and error rate gives rise to much more heat dissipation than expected. Thus, enhancing the cooling power or heat tolerance of a memory device to maintain a proper device temperature is more critical when computation becomes more irreversible. Our formula is also directly applicable for estimating the proper bound of cooling power for a given computation speed, and the density of memory. Furthermore, to save thermodynamic costs, it is important to reduce the dissipation produced inside the same coarse-grained state. Subsequent experimental studies in various physical systems are anticipated in the future.

JSL and SL equally contributed to this work.

Authors acknowledge the Korea Institute for Advanced Study for providing computing resources (KIAS Center for Advanced Computation Linux Cluster System). This research was supported by NRF Grants No. 2017R1D1A1B06035497 (H.P.), and individual KIAS Grants No. PG064901 (J.S.L.), PG081801 (S.L.), CG085301 (H.K.), and QP013601 (H.P.) at the Korea Institute for Advanced Study.

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the coarse-grained bit system.

- [56] Note that for a two-state model, where  $R_{nm}(t)p_m^{ss}(t) = R_{mn}(t)p_n^{ss}(t)$  is satisfied, the Hatano-Sasa EP coincides with the total EP, i.e.  $\Sigma_{\text{HS}} = \Sigma$ ; it can be easily checked by plugging in  $R_{mn}^*(t) = R_{nm}(t) \left(\frac{p_m^{ss}(t)}{p_n^{ss}(t)}\right) = R_{mn}(t)$  into Eq. (2).
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