

Relaxation dynamics of an elastic string in random media

Jae Dong Noh (노재동)

Department of Physics, University of Seoul, Seoul 130-743, Korea

Hyunggyu Park (박형규)

School of Physics, Korea Institute for Advanced Study, Seoul 130-722, Korea

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We investigate numerically the relaxation dynamics of an elastic string in two-dimensional random media by thermal fluctuations starting from a flat configuration. Measuring spatial fluctuations of its mean position, we find that the correlation length grows in time asymptotically as $\xi \sim (\ln t)^{1/\bar{\chi}}$. This implies that the relaxation dynamics is driven by thermal activations over random energy barriers which scale as $E_B(\ell) \sim \ell^{\bar{\chi}}$ with a length scale ℓ . Numerical data strongly suggest that the energy barrier exponent $\bar{\chi}$ is identical to the energy fluctuation exponent $\chi=1/3$. We also find that there exists a long transient regime, where the correlation length follows a power-law dynamics as $\xi \sim t^{1/z}$ with a nonuniversal dynamic exponent z . The origin of the transient scaling behavior is discussed in the context of the relaxation dynamics on finite ramified clusters of disorder.

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Interaction and quenched disorders are essential ingredients in condensed-matter physics. Many-body systems may undergo a phase transition as a cooperative phenomenon mediated by interaction. When quenched disorder comes into play, there may emerge a glass phase in which degrees of freedom are pinned by random impurities and a slow relaxation dynamics appears. An elastic string in random media is one of the simplest systems where the interplay between interaction and quenched disorder yields a nontrivial effect [1,2]. This has been studied extensively in literatures since it is relevant to many interesting physical systems such as a growing interface [3], a domain wall in random magnets [4,5], and a magnetic flux line in superconductors [6].

In low temperatures, one may approximate an elastic string as an elastically coupled directed polymer where no overhang is allowed. Then it can be described by a single valued function $\mathbf{x}(u)$, where $\mathbf{x} \in \mathcal{R}^d$ and $u \in \mathcal{R}$ are the transverse and the longitudinal coordinates to the polymer direction, respectively, in a $(d+1)$ dimensional space. The energy of a polymer of length L in a configuration $\mathbf{x}(u)$ with $0 \leq u \leq L$ is given by the Hamiltonian

$$\mathcal{H} = \int_0^L du \left[\frac{1}{2} \left| \frac{\partial \mathbf{x}}{\partial u} \right|^2 + V(u, \mathbf{x}(u)) \right]. \quad (1)$$

The first term accounts for an elastic tension and the second term $V(u, \mathbf{x})$ is a random pinning potential with short-range correlations.

Equilibrium properties of the directed polymer in random media (DPRM) are rather well understood. The tension favors a flat state, while thermal fluctuations and the disorder potential favor a rough state. The competition between them leads to the scaling law $|\Delta \mathbf{x}| \sim L^\zeta$ for the transverse fluctuation (interface roughness) and $\Delta E \sim L^\chi$ for the (free) energy fluctuation. The quenched disorder is relevant for $d \leq 2$, and the polymer is in a super-rough phase ($\zeta > 1/2$) at all temperatures. Especially for $d=1$, the scaling exponents are known exactly as $\zeta_{1D}=2/3$ and $\chi_{1D}=1/3$ [4]. For $d > 2$, it is believed that there is a transition from a super-rough phase

into a normal-rough phase ($\zeta=1/2$) as the temperature T increases [2]. In the latter, the thermal fluctuations dominate over the disorder fluctuations, while vice versa in the former.

When a polymer is in a nonequilibrium state, e.g., a flat configuration, it will relax to the equilibrium rough state. Without disorder or in the normal-rough phase with disorder for $d > 2$, the elastic polymer equilibrates diffusively. The correlation length ξ in the longitudinal u direction grows algebraically in time as $\xi \sim t^{1/z_0}$ with the dynamic exponent $z_0=2$. In the presence of the quenched disorder, one expects a slower relaxation because random impurities tend to trap the polymer into metastable states in local energy valleys. Upon equilibration, the polymer has to overcome energy barriers E_B separating those valleys through thermal fluctuations to approach the true equilibrium state. It is believed that the energy barrier height scales as $E_B(\ell) \sim \ell^{\bar{\chi}}$ in a region with linear size ℓ .

Thermal activations allow the correlated polymer segment of length ξ to overcome the energy barriers in a time scale $t_\xi \sim e^{E_B(\xi)/T}$. Then, it follows that the correlation length ξ grows as

$$\xi(t) \sim (T \ln t)^{1/\bar{\chi}}, \quad (2)$$

with the universal energy barrier exponent $\bar{\chi}$, independent of the disorder strength and the temperature [4]. Assuming that there is only a single relevant energy scale in this system, the exponent $\bar{\chi}$ should be equal to the energy fluctuation exponent χ [4]. This conjecture is supported at least in low dimensions [7].

However, even for $d=1$, there is a long-standing controversy on the scaling law of Eq. (2). Numerical simulation study [8] reports a signature of the expected logarithmic scaling but only after a long and clean intermediate power-law scaling regime where $\xi \sim t^{1/z}$ with a nonuniversal dynamic exponent z , whose origin is not clear. Moreover, the scaling exponent associated with the logarithmic scaling seems different from the conjectured value of $\bar{\chi}=\chi=1/3$ [8]. There is also a recent claim of $\bar{\chi}=d/2$ based on the droplet theory [9].

Besides, there are many numerical works in the context of domain-wall coarsening dynamics in two-dimensional random ferromagnets [5], which seem to support the nonuniversal power-law scaling without any signature of the asymptotic logarithmic scaling [10,11]. These intriguing results are also left unexplained.

In this work, we study the relaxation dynamics of the DPRM in (1+1) dimensional lattices with extensive numerical simulations and a scaling theory approach. Measured are the spatial fluctuations of the mean position of the polymer, from which we derive the dynamic scaling behavior of the correlation length $\xi(t)$ through a simple scaling hypothesis. The purpose of this study is to settle down the controversy by providing a decisive numerical evidence on the asymptotic relaxation dynamics of the DPRM for $d=1$. In addition, we suggest a reasonable scenario for the origin of the transient power-law scaling regime.

We consider a discrete model for the DPRM in the 45° rotated square lattices of size $L \times M$. Each lattice site is represented as (i, x) with the longitudinal coordinate $i = 0, \dots, L-1$ and the transverse coordinate $x = -M/2 + 1, \dots, M/2$ with the constraint $i=x$ in modulo 2. Assigned to bonds are quenched disorder variables J which are distributed independently and randomly according to a probability density function $p(J)$. The polymer of length L is placed along the bonds and directed in the longitudinal direction without any back bending. Then its configuration is described by the fluctuating variables $\{x(i)\}$ with the solid-on-solid (SOS) constraint of $|x(i) - x(i \pm 1)| = 1$ for all i . We adopt the periodic boundary condition in the longitudinal direction, i.e., $x(L) = x(0)$. The transverse size M is taken to be large enough ($M = 4096 - 8192$) to avoid any possible interference.

The polymer energy is given by the lattice Hamiltonian

$$\mathcal{H} = \sum_{i=1}^L J(i, x(i); i+1, x(i+1)), \quad (3)$$

where $J(i, x; i+1, x')$ denotes the disorder strength of the bond between neighboring sites (i, x) and $(i+1, x')$. In this Rapid Communication, we consider the uniform distribution in the range $-1 \leq J \leq 1$ and the bimodal distribution $p(J) = f\delta(J+1/2) + (1-f)\delta(J-1/2)$ with a model parameter f . It turns out that both cases lead to the same conclusion.

We start with the flat configuration with $\{x(i) = i \bmod 2\}$ at $t=0$ and study its relaxation dynamics toward the equilibrium state at temperature T . We adopt the Glauber Monte Carlo dynamics: first select a site i at random and try to flip $x(i) \rightarrow x(i) \pm 2$ with probability $1/2$, respectively. Unless the trial violates the SOS constraint, it is accepted with the probability of $\min[1, e^{-\Delta E/T}]$, where ΔE is the energy change. The time is incremented by one unit after L such trials.

We focus on the spatial dispersion of the mean position $\bar{x} \equiv \sum_i x(i)/L$ as function of elapsed time t , which is given as

$$(\Delta x)^2(t) \equiv [(\bar{x}(t) - \bar{x}(0))^2]_T \rangle_D. \quad (4)$$

Here $\langle \cdot \rangle_T$ and $[\cdot]_D$ denote the thermal and disorder average, respectively. Let $\xi(t)$ be the characteristic correlation length of the polymer in the longitudinal direction. At time t , each

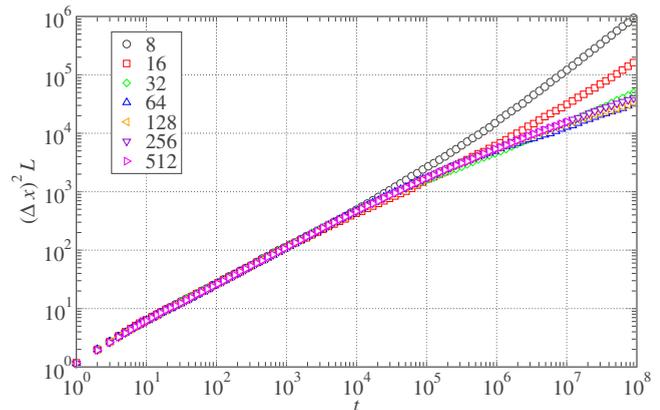


FIG. 1. (Color online) Monte Carlo simulation data for the system with the bimodal disorder distribution with $f=0.25$ and at $T=1.0$. Different symbols represent data for different polymer lengths L . The data are averaged over $N_S=5000$ disorder samples.

segment of size $\xi(t)$ equilibrates with transverse displacement of the order $\delta x \sim \xi^\zeta$ with the roughness exponent ζ . When $\xi \ll L$, each segment is independent and the total displacement is given by

$$(\Delta x)^2 \sim \frac{\xi^{2\zeta}}{(L/\xi)} \sim \frac{\xi^{1+2\zeta}}{L}. \quad (5)$$

Utilizing this relation, we can derive the correlation length $\xi(t)$ from the ensemble-averaged global quantity $(\Delta x)^2(t)$, which usually bears a better statistics than the distance-dependent correlation function of the transverse displacement.

When the polymer fully equilibrates, i.e., $\xi(t=\tau) \simeq L$ with the relaxation time $\tau = \tau(L)$, the polymer as a whole (the mean position) starts to diffuse normally in the transverse direction. One expects that $(\Delta x)^2$ grows linearly in time scaled by τ as

$$(\Delta x)^2 \sim L^{2\zeta} \left(\frac{t}{\tau} \right). \quad (6)$$

Without disorder, the motion of the polymer is governed by the linear Edward-Wilkinson (EW) equation [12]. The EW class is characterized by $\zeta=1/2$ and $\xi \sim t^{1/2}$, i.e., $\tau \sim L^2$. In this case, the dynamic behaviors before and after equilibration [Eqs. (5) and (6)] follow the same scaling law $(\Delta x)^2 \sim t/L$ at all t . Indeed, this coincides with the exact solution of the EW equation. However, in general with disorder, these two scaling laws are distinct.

We have performed extensive Monte Carlo simulations to examine the scaling property of $(\Delta x)^2$. Figure 1 shows a plot of the numerical data with the bimodal disorder distribution with $f=0.25$ at $T=1.0$. As the scaling form predicts in Eq. (5), $(\Delta x)^2$ is inversely proportional to L for $t < \tau(L)$ so it is convenient to plot $L(\Delta x)^2$ versus t , where all curves with different L collapse into one scaling curve for $t < \tau(L)$ and then start to deviate and show the finite size effects given by Eq. (6).

We find that there exist four distinct regimes: (I) for $t < t_0 (\sim 10^1)$, the polymer moves diffusively as $L(\Delta x)^2 \sim t$. In

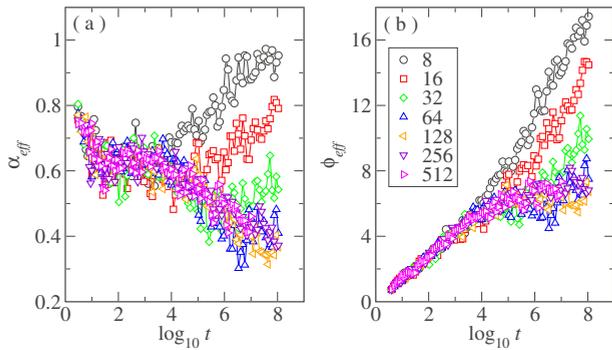


FIG. 2. (Color online) Effective exponent plots for the data given in Fig. 1. Our estimates are $\alpha=0.63 \pm 0.03$ in regime II and $\phi=6.8 \pm 0.5$ in regime III.

this regime, the polymer behaves as in the EW class since it does not feel the disorder pinning as yet. (II) For $t_0 < t < t_c (\sim 10^4)$, the polymer is affected by the disorder and exhibits a power-law scaling behavior as $L(\Delta x)^2 \sim t^\alpha$ with a nonuniversal exponent α . The crossover time t_c is very large but finite and independent of L , which implies that this power-law scaling is transient. (III) For $t_c < t < \tau(L)$, there is a continuous downward curvature in the plot suggesting a possible logarithmic scaling with $L(\Delta x)^2 \sim (\ln t)^\phi$. The crossover time $\tau(L)$ increases indefinitely with L , which implies that this regime should be the true asymptotic scaling regime. (IV) For $t > \tau(L)$, the polymer displays a diffusive motion with a size-dependent diffusion amplitude as in Eq. (6).

We investigate the scaling behavior in each regime quantitatively. Useful are the effective exponents defined as $\alpha_{eff}(t) \equiv d \ln[L(\Delta x)^2] / d \ln t$ and $\phi_{eff}(t) \equiv d \ln[L(\Delta x)^2] / d \ln \ln t$. If $L(\Delta x)^2 \sim t^\alpha$ as in the regime II, one would obtain that $\alpha_{eff}(t) = \alpha$ and $\phi_{eff}(t) = \alpha \ln t$. On the other hand, if $L(\Delta x)^2 \sim (\ln t)^\phi$ as in the regime III, one would obtain that $\alpha_{eff}(t) = \phi / \ln t$ and $\phi_{eff}(t) = \phi$.

We plot the effective exponents in Figs. 2(a) and 2(b). In (a), one can clearly see a plateau at $\alpha_{eff} = 0.63 \pm 0.03$ for $10^1 \leq t \leq 10^4 (=t_c)$ (regime II). For $t > t_c$, it continuously decreases in the regime III before hiking up in the regime IV as expected. In (b), there appears a plateau at $\phi_{eff} = 6.8 \pm 0.5$ in the regime III, which widens as L increases. These numerical evidences lead to a definitive conclusion that the asymptotic motion of the polymer follows the logarithmic (not power-law) scaling as

$$L(\Delta x)^2(t) \sim (\ln t)^\phi, \quad (7)$$

with the exponent $\phi = 6.8 \pm 0.5$. In terms of the correlation length, we find, using Eq. (5),

$$\xi(t) \sim (\ln t)^{1/\tilde{\chi}}, \quad (8)$$

where $1/\tilde{\chi} = \phi / (1 + 2\zeta) = 2.9 \pm 0.2$. The estimated value of $\tilde{\chi} = 0.34 \pm 0.03$ clearly favors the conjecture value of $\tilde{\chi} = \chi = 1/3$ and invalidates the recent claim of $\tilde{\chi} = 1/2$ (or equivalently $\phi = 14/3$).

In order to examine the universality of the logarithmic scaling behavior, we have performed the simulations with different values of f in the case of the bimodal disorder dis-

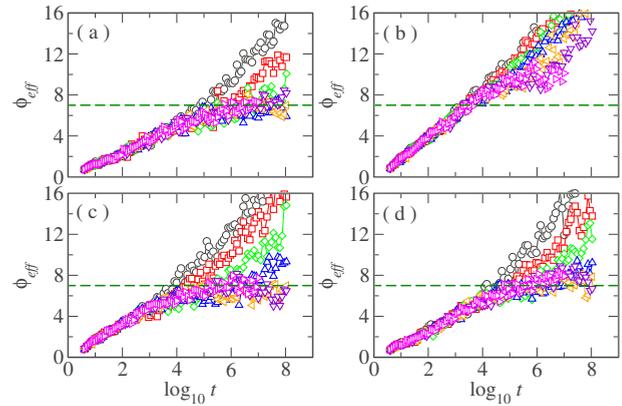


FIG. 3. (Color online) Effective exponent plots with the uniform disorder distributions with the temperature $T=0.25$ in (a) and $T=0.75$ in (b), and the bimodal disorder distribution with $f=0.1$ and $T=1$ in (c) and $f=0.5$ and $T=1$ in (d). The symbols have the same attribute as in Fig. 1. The dashed lines are drawn at $\phi_{eff} = 7.0$.

tribution and also in the case of the uniform disorder distribution. The effective exponents are then presented in Fig. 3. The plateaus around $\phi_{eff} = 7.0$ shown in Figs. 3(a), 3(c), and 3(d) confirm the universality of the logarithmic scaling as well as the universality of its exponent $\tilde{\chi}$. The plot in Fig. 3(b) seems to be incompatible with $\phi = 7$. Comparing Figs. 3(a) and 3(b), one can notice that the finite-size behavior (regime IV) sets in earlier at the higher temperature. So the logarithmic scaling regime is observed only when $L \geq 256$ in (b), while it is already evident at $L = 64$ in the other cases. This suggests that one would need larger polymers at higher temperatures.

It is puzzling why there exists the extremely long transient regime II where the polymer relaxation seems to follow a power-law scaling such as $L(\Delta x)^2 \sim t^\alpha$ or $\xi \sim t^{1/z}$, with $z = (1 + 2\zeta) / \alpha$. Moreover the exponent α is nonuniversal and varies with the disorder strength and the temperature. Such a transient behavior was also reported in Refs. [8,13], but its origin has never been explored. We suggest one reasonable scenario as below.

It is convenient to consider the bimodal disorder distribution. When $f < 1/2$, the energetically favorable bonds with $J = -1$ may play the role of local pinning centers for the polymer [14]. As the polymer considered here is directed, relevant are the directed percolation clusters of the pinning bonds. These clusters are ramified but finite in size, as the bond density f is smaller than the directed percolation threshold $f_c \approx 0.6449$ in the square lattice [15]. The characteristic size and the mean distance between them are denoted by l_0 and l_1 , respectively. After the initial diffusive motion, polymer segments are trapped by those clusters independently as long as the correlation length is smaller than the cluster size ($\xi < l_0$).

The pinning mechanism in fractal-like ramified lattices is different from that in bulks. The energy barrier height in such lattices is shown to scale logarithmically with a length scale ℓ as $E_B(\ell) \approx E_0 \ln \ell$ with an universal constant E_0 depending only on the ramification degree [16]. Such a logarithmic barrier has also been observed in the so-called marginal glass phase [17]. Then, the time scale associated with the thermal

activation of the correlated segment of length ξ is given by $t_\xi \sim e^{E_B(\xi)/T} \sim \xi^{E_0/T}$. This yields the power-law growth of the correlation length as $\xi \sim \sqrt{t/t_\xi} \sim t^{1/z}$ with the nonuniversal dynamic exponent $z=2+E_0/T$. The temperature dependence seems consistent with our numerical estimates for z (not shown here). When ξ exceeds l_0 , the polymer segments are pinned by a few pinning clusters. If $\xi \geq l_1$, then the polymer starts to be pinned collectively, and the transient power-law scaling behavior crosses over to the asymptotic logarithmic scaling behavior.

Finally, we add one remark on the domain coarsening dynamics in the two-dimensional random ferromagnets. When the system is quenched well below an ordering temperature from a disordered state, the characteristic size R of ordered domains increases and the domain-wall motion may be described by the DPRM. Hence, it is natural to expect that $R(t) \sim \xi^{2-\xi} \sim (\ln t)^{(2-\xi)/\bar{\chi}}$ [4]. Surprisingly, recent high accuracy numerical simulation studies report that $R(t) \sim t^{1/z}$ with a nonuniversal exponent z [11]. Our result suggests that those

behaviors may be due to the pinning of domain walls by finite pinning clusters in the transient regime.

In summary, we have investigated numerically the relaxation dynamics of the DPRM. The numerical data show unambiguously that the correlation length grows as $\xi \sim t^{1/z}$ in the transient regime and then $\xi \sim (\ln t)^{1/\bar{\chi}}$ in the asymptotic regime. The transient behavior is originated from the pinning independently by local ramified impurity clusters. The asymptotic logarithmic scaling is compatible with the scaling picture that the energy barrier height scales in the same way as the energy fluctuations with $\bar{\chi}=\chi=1/3$. Implication on the domain coarsening dynamics is also discussed.

Note added in proof. Recently, Iguain *et al.* [18] presented results that partially overlap with those presented here.

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