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Two-Species Branching Annihilating Random Walks with One Offspring

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We study the effects of hard core (HC) interactions between different species of particles on two-species branching annihilating random walks with one offspring [$BAW_2(1)$]. The single-species model belongs to the directed percolation (DP) universality class. In the $BAW_2(1)$ model, a particle creates one particle of the same species in its neighborhood with the probability $\sigma(1-p)$ and of the different species with $(1-\sigma)(1-p)$, where p is the hopping probability. Without HC interactions, this model always exhibits the DP-type absorbing transition for all σ. Even with HC interactions, the nature of the phase transitions does not change except near $\sigma = 0$, where the HC interaction destabilizes and completely wipes away the absorbing phase. The model is always active except at the annihilation fixed point of zero branching rate ($p = 1$). Critical behavior near the annihilation fixed point is characterized by the exponents $\beta = \nu_\perp = 1/2$ and $\nu_{||} = 1$.

The last decades have seen considerable efforts to understand nonequilibrium absorbing phase transitions from an active phase into an absorbing phase consisting of absorbing states [1]. Once the system is trapped into an absorbing state, it can never escape from the state. Various one dimensional lattice models exhibiting absorbing transitions have been studied, and most of them turn out to belong to one of two universality classes, the directed percolation (DP) and the directed Ising (DI) universality classes. While models with the Ising symmetry between absorbing states belong to the DI class, models in the DP class have no symmetry between absorbing states [1,2].

It is generally accepted that the dimensionality and the symmetry between absorbing states play important roles in determining the universality classes as in equilibrium critical phenomena. To find out new universality classes, it is natural to study models with higher symmetries than the Ising symmetry. To achieve a higher symmetry, such as the Potts symmetry, one may increase the number of absorbing states. However, the models with higher symmetries investigated so far turn out to be always active and are only critical at the annihilation fixed point of zero branching rate [3]. A recent field theoretical study may provide a possible explanation for this [4]. Although there is no stable absorbing phase, critical behaviors near the annihilation fixed point are non-trivial and form new universality classes [4,5].

In systems with more than two equivalent absorbing states, there are various kinds of domain walls (or kinks) that cannot cross over each other to retain the order of absorbing domains. Multiple occupations of domain walls at a site is also forbidden. These restrictions may be naturally realized by introducing hard core (HC) interactions between different domain walls. These domain wall dynamics can be considered as a multi-species particle dynamics with HC interactions.

Recently, Cardy and Täuber introduced an interesting multi-species model, N-species branching annihilating random walks with two offsprings (N-BAW(2)), which can be solved exactly for all $N > 1$, by using renormalization group techniques in a bosonic type formulation which ignores HC interactions [4]. The N-BAW(2) model is a classical stochastic system consisting of N species of particles, A_i ($i = 1, \dots, N$). Each particle diffuses on a d-dimensional lattice with two competing dynamic processes: pair annihilation and branching. Pair annihilation is allowed only between identical particles ($A_i + A_i \to \emptyset$). In the branching process, a particle A_i creates two identical particles in its neighborhood ($A_i \to A_i + 2A_j$), with rate σ for $i = j$ and rate $\sigma'/(N-1)$ for $i \neq j$. For $N = 1$, this model exhibits a DI-type absorbing transition at a finite branching rate [6–9]. For $N > 1$, this model is always active, except at the annihilation fixed point of zero branching rate.

Our recent study of the one dimensional N-BAW(2) model shows that the HC interaction between different species of particles drastically changes the universality class in a non-trivial way [5]. This contradicts the conventional belief that the HC interaction is irrelevant to absorbing-type critical phenomena, because the particle density is so low near an absorbing transition that the probability of multiple occupations at a site should be too small to be significant. However, in multi-species

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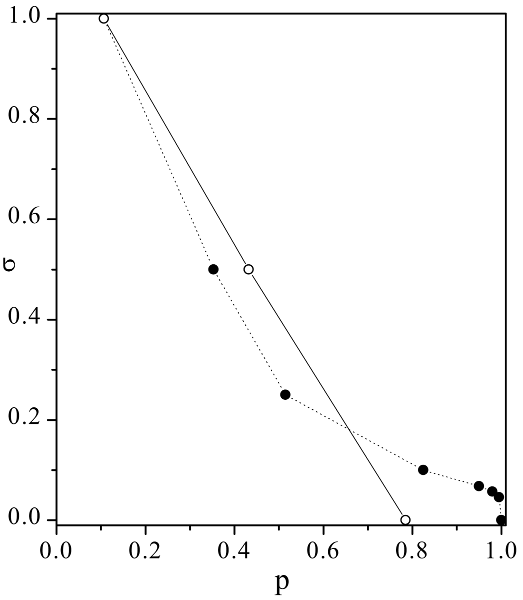

Fig. 1. The $\sigma - p$ phase diagram for the $BAW_2(1)$ model. Open and filled circles correspond to the critical points without and with the hard core interaction, respectively. Lines between data points are guides to the eyes only.

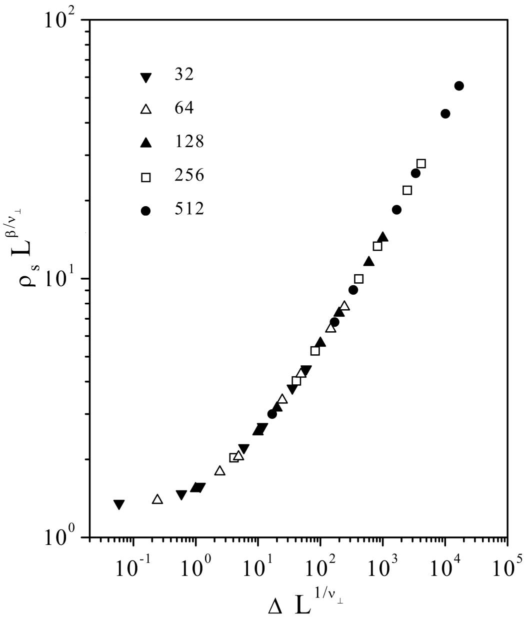

Fig. 2. Data collapse of $\rho_s L^{\beta/\nu_\perp}$ against $\Delta L^{1/\nu_\perp}$ with $\beta/\nu_\perp = 1$ for various system sizes $L = 2^5, \ldots, 2^9$ for $BAW_2(1)$ with the hard core interaction at $\sigma = 0$. Best collapses are achieved with $\nu_\perp = 0.49(4)$.

models of $N > 1$, the asymptotic density decay near the annihilation fixed point becomes nontrivial due to the convective displacement generated by the HC interaction.

In this paper, we study two-species branching annihilating random walks (BAW) with one offspring [$BAW_2(1)$] in one dimension. The single-species BAW model belongs to the DP universality class [6,10]. Thus, this model is a natural multi-species extension of DP-type systems. We numerically study the effect of the HC interaction on the absorbing phase transition and find that the HC interaction changes the phase diagram considerably.

The $BAW_2(1)$ model is a stochastic system consisting of two species of particles, A and B. Each particle hops to a nearest neighbor site with probability p or creates a particle on a nearest neighbor site with probability $1-p$. The created particle can be either of the same species as its parent with probability σ or of the different species with probability $\sigma' = 1 - \sigma$. With HC interactions, any dynamics resulting in multiple occupations at a site by different species of particles is forbidden.

We perform dynamic Monte Carlo simulations. With the initial condition of a nearest neighbor pair of identical particles, we measure the survival probability $P(t)$ (the probability that the system is still active at time t), the number of particles $N(t)$ averaged over all runs, and the mean distance of spreading $R(t)$ averaged over the surviving runs.

At criticality, these quantities scale algebraically in the long time limit as $P(t) \sim t^{-\delta}$, $N(t) \sim t^{\eta}$, and $R(t) \sim t^{1/z}$ [11]. The effective exponents defined as

$$-\delta(t) = \log[P(t)/P(t/b)]/\log b \qquad (1)$$

and similarly for $\eta(t)$ and $1/z(t)$, show straight lines at criticality and upward (downward) curvatures in the active (absorbing) phase. With $b = 5$, we estimate the asymptotic values of dynamic critical exponents δ, η, z and of the critical hopping probability p_c for various values of σ.

Fig. 1 shows the $\sigma - p$ phase diagram of the $BAW_2(1)$ model. Without HC interactions, this model always exhibits the DP-type absorbing transition for all σ. For example, at $\sigma = 0$, we estimate the critical probability as $p_c = 0.785(5)$ and the dynamic scaling exponents as $\delta = 0.155(5)$, $\eta = 0.32(1)$, and $1/z = 0.640(5)$, which agree well with the DP values [7].

Even with HC interactions, the nature of the phase transitions does not change, except near $\sigma = 0$. However, near $\sigma = 0$, the HC interaction destabilizes and completely wipes away the absorbing phase. The model is always active, except at the annihilation fixed point of zero branching rate ($p_c = 1$). Numerically, we could not pinpoint when the absorbing phase is completely squeezed out, due to statistical errors. With present simulations, we estimate the upper bound of σ as $\sigma_c \simeq 0.05$. To identify the scaling behavior near the annihilation fixed point of the system with the HC interaction, we analyze the finite-size effects on the steady-state particle density ρ_s.

The scaling behavior near criticality is characterized by exponents β, $\nu_\perp$, and $\nu_{||}$ defined as

$$\xi \sim \Delta^{-\nu_\perp}, \qquad \tau \sim \xi^z,$$
$$\rho(t) \sim t^{-\alpha}, \qquad \rho_s \sim \Delta^\beta, \tag{2}$$

where $\Delta = p_c - p$, ξ is the correlation length, τ the characteristic time, $\rho(t)$ the particle density at time t, and ρ_s the steady-state particle density. At the annihilation fixed point, the exponents α and z should follow from the simple random walk exponents $\alpha = 1/z$ and $z = 2$ [12].

Using the finite-size scaling theory on the steady-state particle density ρ_s [13]

$$\rho_s(\Delta, L) = L^{-\beta/\nu_\perp} F(\Delta L^{1/\nu_\perp}), \tag{3}$$

the value of $\nu_\perp$ is determined by collapsing the data of ρ_s with $\beta/\nu_\perp = 1$. We measure ρ_s in the steady state, averaged over $5\times10^3 \sim 5\times10^4$ independent samples for several values of Δ ($5\times10^{-4} \sim 0.05$) and lattice size L ($2^5 \sim 2^9$). We find $\nu_\perp = 0.49(4)$ (Fig. 2). With $\alpha = 1/2$ and $z = 2$, we estimate

$$\beta = 1/2, \quad \nu_\perp = 1/2, \quad \nu_{||} = 1. \tag{4}$$

The single-species BAW(1) model is one of the simplest models belonging to the DP class. BAW(1) dynamics involves the spontaneous annihilation of a particle ($A \to \emptyset$), which is a common feature of DP models. A single particle is spontaneously annihilated by the combination of a branching (creation) and hopping process, such as $A \to A + A \to \emptyset$.

As σ decreases, the pair annihilation by diffusion seldom occurs because the collision probability of the same species of particles decreases. It makes the system more active, so the critical hopping probability(p_c) must be an increasing function of $\sigma' = 1-\sigma$. This is consistent with our numerical phase diagram (Fig. 1).

The effect of the HC interaction on the phase diagram turns out to be rather tricky. For large σ, the system tends to form large A- or B-dominated domains. The HC interaction accelerates the pair annihilation process because the domain boundary will induce diffusion bias to the center of each domain. Therefore, the system becomes less active, and the absorbing phase expands with the HC interaction.

However, for small σ, the above story is completely reversed. The system tends to have a high density of locally ordered AB pairs. Without the HC interaction, most of pair annihilation processes should occur by diffusions across the domain boundary. Thus, in this case, the HC interaction decreases the chance of pair annihilation processes, and the system becomes more active. This implies that the critical lines with and without HC interactions should cross each other, as in Fig. 1.

At $\sigma = 0$, a single particle cannot be spontaneously annihilated by the combination of a single branching process and diffusions. It needs at least three branching processes, *e.g.*, $A \to AB \to ABA \to ABAB$, which can be annihilated by successive diffusions in systems without HC interactions. Hence, the absorbing phase is still stable, and the DP nature is maintained.

However, with HC interactions, the ordered AB pairs can not be annihilated by diffusions. A single particle has a finite probability to survive asymptotically, and the entire absorbing phase becomes unstable. The system becomes always active, except at the annihilation fixed point of zero branching rate.

Critical behavior near the annihilation fixed point can be extracted by balancing the particle density changes due to branching and pair annihilation processes. Let us consider a particle A created by a particle B at time t. Annihilation of the created particle A occurs by encountering an independent A particle through diffusions. The time scale of this pair annihilation process is given by ordinary diffusions, *i.e.*, $\tau_d \sim d^2$ where the mean distance between particles d is order of $\rho(t)^{-1}$. Branching process of the parent particle B increases the particle density with the time scale $\tau_b \sim (1-p)^{-1}$. By balancing these two time scales, we expect the steady-state particle density to scale as $\rho_s \sim (1-p)^\beta$ with $\beta = 1/2$. This is consistent with our numerical finding in Eq. (4).

The above argument should be valid for general N-species BAW(1) models for $N \geq 2$ near the annihilation fixed point. Preliminary numerical results for $N = 3$ confirm our predictions. The universality class characterized by exponents $\beta = \nu_\perp = 1/2$ and $\nu_{||} = 1$ also includes the N-species BAW(2) model for $N \geq 2$ with static branching processes, where two offsprings are divided by their parent, such as $\emptyset B \emptyset \to ABA$ [5]. It is clear that our argument above also applies to this model.

In summary, we studied the two-species branching annihilating random walks with one offspring. This model exhibits a DP-type absorbing phase transition as in the single-species model. With hard core interactions between different species of particles, the phase diagram changes considerably. Especially, the system becomes always active with an annihilation fixed point for a very small branching rate of the same species. We find that the critical behavior near the annihilation fixed point is identical to that in the N-BAW(2) model with hard core interactions and static branching. The N-species generalization of the BAW(1) model is currently under study.

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