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Relaxation of the Non-Order Parameter Field in Directed Ising Systems

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We investigate the effect of initial conditions on the dynamic exponents of the interacting monomer-monomer model with infinitely many absorbing states in one dimension. This model exhibits a directed Ising (DI) type transition from an active phase into an absorbing phase. For the directed percolation universality class, the non-order parameter was reported to exhibit critical fluctuations, relaxing algebraically to its natural value with the same scaling exponents as in the order parameter . We numerically confirm that this is also valid for the DI universality class. We also observe continuously varying dynamic exponents with a linear dependence on the non-order parameter initial density.

Various kinds of nonequilibrium lattice models exhibiting absorbing phase transitions have been studied extensively during the last few decades [1]. Two distinct types of absorbing phase transitions have been identified in one dimension: the directed percolation (DP) and directed Ising (DI) universality classes. Most models have been found to belong to the DP class, which involves typically a single absorbing state or multiple absorbing states without any symmetry. DI-type critical behavior appears in models with two equivalent absorbing states or two equivalent groups of absorbing states [2–4].

Recently, the question whether one can construct initial states that affect the *entire* temporal evolution of nonequilibrium systems, including asymptotic dynamics, has been addressed. Typical examples are systems that display an absorbing phase transition from an active phase into an absorbing phase with infinitely many absorbing (IMA) states. As in the ordinary order-disorder phase transition models, the order parameter takes a nonzero value in the active phase and vanishes in the absorbing phase. The IMA states can be characterized by a *non-order* parameter. In the steady state, the non-order parameter approaches algebraically to a *natural* value at criticality.

Nonuniversal dynamic properties have been reported in various kinds of DP-type nonequilibrium lattice models with IMA states [5,6]. The order-parameter dynamic exponents vary continuously with the initial conditions characterized by the non-order parameter density. Moreover, at criticality, the non-order parameter density also exhibits critical fluctuations, relaxing algebraically to its natural value with the same dynamic exponents [7]. The critical relaxation of the non-order parameter field in DP-type systems with IMA states was recently examined from a field theoretical (Langevin equation) approach [8,9]. The evolution equation for the order parameter is non-Markovian and includes a temporal memory term due to the non-order parameter field. This non-Markovian term is known to be responsible for the nonuniversal dynamic exponents.

In this paper, we introduce an interacting monomer-monomer model with IMA states (IMA-IMM model) that belongs to the DI universality class and investigate the effects of the non-order parameter field in one dimension. Using numerical simulations, we show that the non-order parameter plays the same role in both DI and DP systems.

The IMA-IMM model is an interacting monomer-monomer model with two different species of monomers, namely A and B. Monomers A and B are selected with probabilities, p and $1-p$, respectively. Monomers can adsorb at a randomly selected vacant site with unit probability for A and with probability $1-r_B$ for B. A monomer adsorption attempt is rejected when both sites adjacent to a selected vacant site are occupied or at least one adjacent site is occupied with a monomer of the same species. A nearest neighbor AB pair reacts and desorbs immediately from the lattice:

$$\begin{aligned} A+V &\xrightarrow{p} A_s, \\ B+V &\xrightarrow{(1-p)(1-r_b)} B_s, \\ A_s+B_s &\longrightarrow \emptyset. \end{aligned} \tag{1}$$

Here, the subscript s denotes adsorbed particles, and V denotes a vacant site where the adsorption attempt is allowed.

This model has infinitely many absorbing states. Any configuration without a nearest neighbor pair of vacant sites is an absorbing state. The number of absorbing

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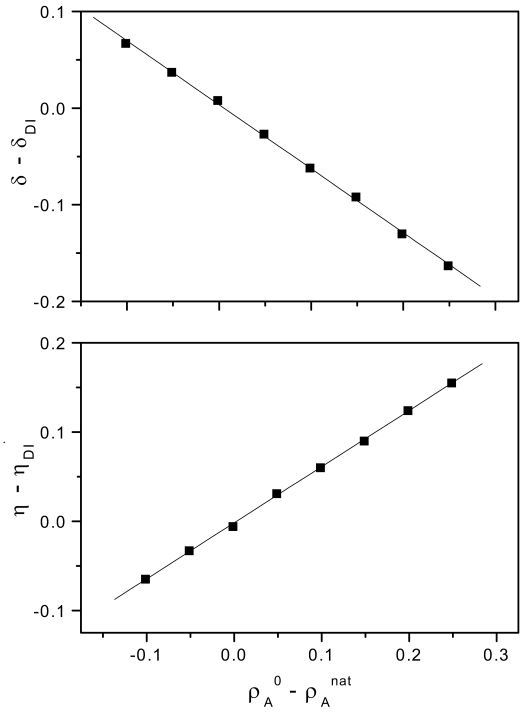

Fig. 1. Temporal dependence of $\Delta\rho_A$ and size dependence of $\Delta\rho_A$ and τ_A at $p = p_{c1}$. The straight lines have slopes of $0.26(=\beta/\nu_\parallel)$, $0.46(=\beta/\nu_\perp)$, and $1.80(=\nu_\parallel/\nu_\perp)$.

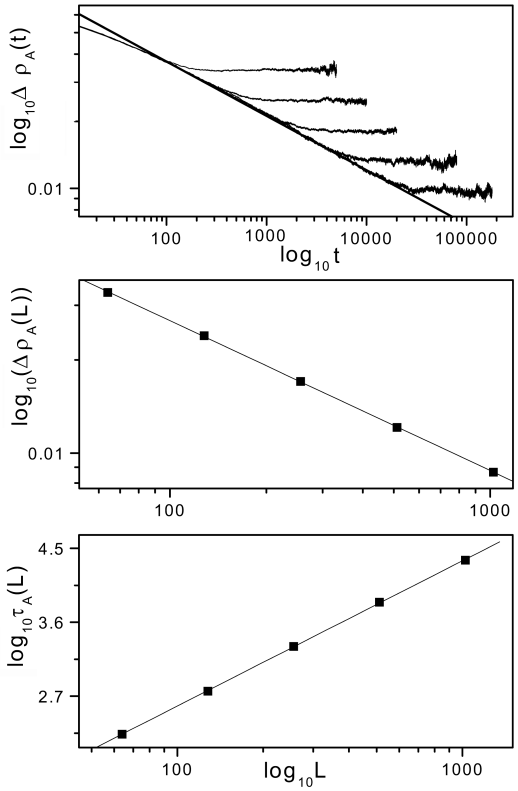

Fig. 2. Initial configuration dependence of the exponents δ and η at $p = p_{c1}$. A linear dependence on the initial non-order parameter density is observed.

states diverges exponentially with system size L as $2^{L/2}$. The absorbing states can be divided into two equivalent groups, odd-site occupied and even-site occupied groups. Clearly, these two groups of absorbing states have a one-to-one correspondence and the Ising (Z_2) symmetry in between them. Thus, we expect this model to exhibit the DI-type absorbing phase transition.

We locate the critical line in the $r_b - p$ phase diagram by using dynamic Monte Carlo simulations [10]. For small values of r_b, the system is always absorbing. As r_b increases, a window of the active phase appears and divides the absorbing phase into A-dominated and B-dominated absorbing phases. For example, as p increases along the $r_b = 0.9$ line, the system undergoes two continuous phase transitions from the B-dominated absorbing phase into the active phase at $p_{c1} = 0.100(3)$ and finally into the A-dominated absorbing phase at $p_{c2} = 0.5058(7)$. We numerically confirm that both transitions are of the DI type, as expected [10].

The order parameter ρ is the number density of nearest neighbor pairs of vacant sites, while we define the A particle density as the non-order parameter ρ_A. First, we perform static Monte Carlo simulations to measure the natural density of the non-order parameter at both criticalities. We measure the A particle density, $\rho_A(t, L)$, averaged over $2 \times 10^3 \sim 5 \times 10^4$ survived samples for system size $L = 2^6 \sim 2^{10}$. $\rho_A(t, L)$ relaxes to the natural density $\rho_A^{nat} \equiv \rho_A(\infty, \infty)$ in the thermodynamic limit. At $p = p_{c1}$ and $p = p_{c2}$, we estimate $\rho_A^{nat} = 0.101(1)$ and $0.458(2)$, respectively. The distribution function for the A particle density appears to be Gaussian, so the mean density is identical to the most probable density. We measure the most probable density in the long time limit, which turns out to be consistent with the above mean natural density within statistical errors.

Dynamic properties for the non-order parameter $\rho_A(t, L)$ can be extracted by studying its temporal deviation from its steady-state value (natural density) as

$$\Delta\rho_A(t, L) \equiv |\rho_A(t, L) - \rho_A^{nat}|. \tag{2}$$

As in the DP case, we assume that $\Delta\rho_A(t, L)$ follows the same scaling behavior as the order parameter. With the finite-scaling theory [11] in the steady state, $\Delta\rho_A(t, L)$ scales at criticality as

$$\Delta\rho_A(\infty, L) \sim L^{-\beta/\nu_\perp}. \tag{3}$$

One can also expect the critical short time behavior to be

$$\Delta\rho_A(t, \infty) \sim t^{-\beta/\nu_\parallel}, \tag{4}$$

and the characteristic time $\tau_A(L)$ to scale as

$$\tau_A(L) \sim L^{\nu_\parallel/\nu_\perp}. \tag{5}$$

Table 1. Initial configuration dependence of the dynamic exponents at $p_c = 0.1$.

ρ_A^0	δ	η
0.00	0.352(5)	−0.065(10)
0.05	0.322(5)	−0.033(7)
0.10	0.293(4)	−0.006(8)
0.15	0.258(8)	0.031(9)
0.20	0.223(4)	0.060(7)
0.25	0.193(5)	0.090(7)
0.30	0.155(3)	0.124(6)
0.35	0.122(4)	0.155(8)

From static simulations, we estimate the scaling exponents at both criticalities. At $p = p_{c1}, p_{c2}$, we estimate $\beta/\nu_\perp = 0.46(2), 0.46(3)$, $\beta/\nu_\parallel = 0.26(2), 0.25(2)$, and $\nu_\parallel/\nu_\perp = 1.75(5), 1.90(10)$ (see Fig. 1). As expected, these estimates involve rather large statistical and systematic errors, especially due to inaccuracies in the natural density values. However, these values agree reasonably well with the DI values [12], which confirms our assumption that the non-order parameter exhibits the same type of critical fluctuations as the order parameter.

The dynamic exponents for the order parameter in DP systems with IMA states are known to depend on initial conditions characterized by the non-order parameter. To investigate the nonuniversal dynamic properties in DI systems, we perform dynamic Monte Carlo simulations with various initial conditions. We start with a pair of nearest neighbor vacant sites in the absorbing background which is controlled by the A particle density ρ_A^0.

We measure the survival probability $P(t)$(the probability that the system is still active at time t) and the mean number of pairs of vacant sites (order parameter), $N(t)$, averaged over all samples. At criticality, these quantities scale algebraically in the long time limit as [13]

$$P(t) \sim t^{-\delta}, \qquad N(t) \sim t^{\eta}. \tag{6}$$

The dynamic exponents are, in general, functions of initial non-order parameter density; $\delta = \delta(\rho_A^0)$ and $\eta = \eta(\rho_A^0)$. For initial configurations of the natural non-order parameter density, the exponents take the ordinary DI values; $\delta(\rho_A^{nat}) = \delta_{DI} \simeq 0.285$ and $\eta(\rho_A^{nat}) = \eta_{DI} \simeq 0.00$.

At $p = p_{c1}$ with $r_b = 0.9$, we estimate the values of δ and η as ρ_A^0 varies from 0 to 0.35 (see Table 1). In Fig. 2, we plot the exponent shifts from the DI values ($\delta - \delta_{DI}$ and $\eta - \eta_{DI}$) versus $\rho_A^0 - \rho_A^{nat}$. It shows a linear dependence of the exponent shifts on the deviation of the non-order parameter from the natural density in initial configurations. This linear dependence has also been seen in DP systems. In summary, we investigated the IMA-IMM model in one dimension, which showed the DI-type continuous phase transition from an active phase into an absorbing phase consisting of infinitely many absorbing states. We found that the non-order parameter exhibits critical fluctuations identical to the order parameter. The dynamic exponents δ and η depend linearly on the initial non-order parameter density and coincide with the ordinary DI values only at the natural density.

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