Nonequilibrium dynamical systems with two absorbing states

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We study the interacting monomer-dimer model with two absorbing states. This model exhibits a continuous phase transition from an active phase into an absorbing phase which belongs to the universality class of the models with the parity conservation. When a symmetry breaking field is introduced, the system goes back to the conventional directed percolation universality class. We perform Monte Carlo simulations for the interface dynamics between different absorbing states. With the symmetry breaking field, the interface moves, in average, with a constant velocity in the direction of the unpreferred absorbing state and the dynamic scaling exponents apparently assume trivial values. However, we find that the hyperscaling relation for the directed percolation universality class is restored if one considers the interface on the side of the preferred absorbing state only.

Nonequilibrium dynamical models may supply much richer critical behavior because their evolving dynamics do not require the detailed balance. So the universality classes of nonequilibrium critical phenomena would be much more diverse and would be governed by various symmetric properties of the evolution dynamics. An interesting example of nonequilibrium phase transitions is the absorbing phase transition from an active phase into an absorbing phase which consists of some absorbing states. Once the system enters into one of the absorbing states, the system is trapped inside the absorbing phase and no further dynamics occur to escape.

Recently, various kinds of nonequilibrium models exhibiting such an absorbing phase transition have been studied extensively. Most of the models investigated are found to belong to the directed percolation (DP) universality class [1–3]. A common feature of these models is that the absorbing phase consists of a single absorbing state.

Only a few models have been studied that are not in the DP universality class. Those are the models A and B of probabilistic cellular automata [4,5], nonequilibrium kinetic Ising models with two different dynamics [6,7], and interacting monomer-dimer models (IMD) [8,9]. It has been found numerically that the critical behaviors of these models are different from DP but form a non-DP universality class. These models share a common property that the absorbing phase consists of two equivalent absorbing states. By the analogy to the equilibrium Ising model, we call this non-DP universality class as the *directed Ising* (DI) universality class. We recently showed for the IMD model that an external field which breaks the symmetry between two absorbing states forces the system back to the conventional DP universality class [10].

In this paper, we perform dynamic Monte Carlo simulations for the interface dynamics of the IMD model with the symmetry-breaking field in one dimension. One of two absorbing states is preferred to the other by the evolution rules. Dynamical behavior of the interface between the preferred and the unpreferred absorbing state is completely different from that without symmetry-breaking field [11,12].

In the active phase, the active region (the width of the interface) grows linearly in time in both directions. Of course, the interface front near the preferred absorbing state (we call it "P-interface front") moves slower than the interface front near the unpreferred absorbing state ("U-interface front") due to the symmetry-breaking

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field. At the critical (transition) point, the P-interface front behaves like a critical interface (zero velocity with nontrivial temporal correlations), but the U-interface front still has a finite velocity. In the absorbing phase, the P-interface front moves with the same finite velocity into the same direction as the U-interface front. So the velocity of the P-interface front has a finite jump at the critical point and the width of the active region between the P-interface and the U-interface front becomes finite in the absorbing phase [12]. In constrast, this width diverges diffusively in the absorbing phase without the symmetry-breaking field. Here, we consider the critical case only.

From the defect dynamics simulations [10,11], we find the critical probability $p_c = 0.4138(3)$ of the monomer adsorption-attempt probability with the symmetry-breaking field $h = 0.5$. At criticality, the survival probability, the number of dimers, and the mean-square distance of spreading scale algebraically in the long time limit with critical exponents δ, η, and z. In the defect dynamics, these values are $\delta = 0.163(5)$, $\eta = 0.315(5)$, and $z = 1.265(5)$ [10,11], which are in an excellent agreement with those of the DP universality class.

We measure these critical exponents for the interface dynamics at criticality. Due to the ballistic nature of the U-interface front even at criticality, these exponents assume trivial values; $\delta = 0$, $\eta = 1$, and $z = 2$. However, the P-interface front behaves in a critical fashion so we concentrate on the profile between the initial position of the interface and the P-interface front only. In order to avoid the confusion when the P-interface front moves across the initial position of the interface, we modify our evolution rule slightly such that we stop simulations and treat that specific run as a dead one when the P-interface front reaches the initial position. In this way, we can define the survival probability and the active region between P-interface front and the initial position. The number of dimers in this region and the mean-square width of this region are measured.

We run 2×10^4 independent samples up to 3×10^4 Monte Carlo time steps at the critical point ($p_c = 0.4138$) with $h = 0.5$. We find that $\delta = 0.37(2)$, $\eta = 0.105(15)$, and $z = 1.27(2)$. As expected, the value of z agrees well with the DP value and these exponents satisfy the generalized hyperscaling relation $2\delta + 2\beta/\nu_{\|} + 2\eta = dz$ [13] for the DP universality class where β and $\nu_{\|}$ are the order-parameter exponent and temporal correlation exponent, respectively, and d is the spatial dimension. Even though the defect and interface dynamics yield different values of δ and η, their sum $\delta + \eta$ which is responsible for the growth of the number of dimers only in surviving clusters should be universal. Here, the sum $\delta + \eta \simeq 0.47$ also agrees with the value of the DP universality class.

In summary, the interface dynamics for the IMD model with the symmetry-breaking field appear to be trivial, but reveal relevant information for the universality class of the absorbing phase transition when one considers the interface on the side of the preferred absorbing state only. This concept may be applied to other reaction models in which many types of interfaces coexist and some of them are not equivalent.

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